Research Seminar of the GRK 1150 Homotopy and Cohomology (WS 2008-2009): The J-homomorphism and the Adams conjecture

Organizers: Johannes Ebert; Gerald Gaudens

Talk 1. *(Gérald Gaudens) Introduction to K-theory and overwiew.*

Talk 2. *(Achim Beckers) Definition of the e-invariant in terms of the Chern character [4], section 7. Omit 7.8 until 7.14. (Remark: Adams denotes it by λ; e has another meaning and later both invariants will be shown to coicide). Include the proof of Theorems 7.15 and 7.16 of [4], which are given in section 10. The discussion depends on the number-theoretic results of [2], section 2 and on some results of [2], section 5 concerning the relationship between the Chern character and the Thom isomorphisms in K-theory and cohomology. A source which might be more accessible is [7], Chapter 4. Hatcher only discusses the complex case (which is both easier and weaker), but his exposition is not mixed up with the other approach to the e-invariant which will be discussed later.*

Talk 3. *Adams operations (Jan Möllers). Introduce the Adams operations ψ^k for both complex and real K-theory and prove their main properties, including their values on spheres. This is done in many places in the literature, for example [5], [6], [7]. Explain the periodicity theorem for Adams operations, [3], Thm 5.1. and give an idea of its proof.*

Talk 4. *(Ferit Deniz) The e-invariant in terms of homological algebra [1], sections 3, 6. Section 3 describes a general framework, which is specialized to K-theory in section 6. Then show that the two definitions of the e-invariant coincide (this is done [4], chapter 7. Also discuss [4], section 9.*

So far, we have established a lower bound on the order of the image of the J-homomorphism. The next two talks will give an upper bound.

Talk 5. *(Christian Ausoni) Spherical fibrations, the Adams conjecture and an upper bound for $Im(J)$. Basic facts about spherical fibrations and interpretation of the J-homomorphism as a forgetful map from vector bundles to spherical fibrations. Introduce the Adams Conjecture (Conjecture 1.2 of [1]), which is now a theorem. Show Theorem 3.7 of [2]; the ambiguity in its statement is solved by the Adams conjecture.*

Talk 6. *(Johannes Ebert) The proof of the Adams conjecture.*

Talk 7. *(Marco Schlichting) Toda brackets and the behaviour of the e-invariant with respect to Toda brackets, [4], sections 4, 5, 11.*

Talk 8. *(Nasko Karamanov) The periodic elements, [4], section 12.*

References

[1] Adams: *On the groups* $J(X)$ *I*

[2] Adams: *On the groups* $J(X)$ *II*

[3] Adams: *On the groups* $J(X)$ *III*

[4] Adams: *On the groups* $J(X)$ *IV*

[5] Adams: *Vector fields on spheres*

[6] Husemoller: *Fibre Bundles*

[7] Hatcher: *Vector bundles and* K*-theory*, available on the author's homepage.