Fourth exercise sheet "Algebra II" winter term 2024/5.

Problem 1 (6 points)**.** *Let K be a field and $R = K[\![T]\!]$ be the ring of all formal power series $f = \sum_{k=0}^{\infty} f_k T^k$. Show that R is a discrete valuation ring.*

An integer will be square-free if it is not divisible by the square of any prime number.

Problem 2 (10 points)**.** *Let $K = \mathbb{Q}\big(\sqrt{D}\big)$ where $D \neq 1$ is a square-free integer. Show that a base of the free abelian group $\mathcal{O}_K$ is given by* 1 *together with*

$$\begin{cases} \frac{1+\sqrt{D}}{2} & D \equiv 1 \bmod 4 \\ \sqrt{D} & \textit{otherwise} \end{cases}$$

Problem 3 (4 points)**.** *Let K be a field, $K^\times \xrightarrow{v} \mathbb{Z}$ a surjective map satisfying*

$$(+) \qquad v(xy) = v(x) + v(y) \qquad\qquad v(x+y) \geq \min\big(v(x), v(y)\big)$$

where we put $v(0) = \infty$. Moreover, let L/K be a finite purely inseparable extension. Show that there is a unique extension $L^\times \to \mathbb{Q}$ of v which also satisfies (+).

Solutions should be submitted to the tutor by e-mail before Friday November 8 24:00.