Minisymposium 5

Finanznumerik
(Computational Finance)

Leiter des Symposiums:

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Dienstag, 19. September

Raum 610, Institut für Angewandte Mathematik, Wegelerstr. 6

15:00 – 15:50 **Kees Oosterlee** *(Delft)*
Fast and Accurate Pricing of Early Exercise Options with the Fast Fourier Transform

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16:00 – 16:50 **Ansgar Jüngel** *(Mainz)*
Nonlinear Black-Scholes-type Equations for Financial Derivatives

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17:00 – 17:20 **Ralf Forster** *(FU Berlin)*
Fast and Reliable Pricing of American Options With Local Volatility

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17:30 – 17:50 **Sebastian Quecke** *(Köln)*
Pricing American Options using Quadrature

Mittwoch, 20. September

Raum 610, Institut für Angewandte Mathematik, Wegelerstr. 6

15:00 – 15:50 **Erich W. Farkas** *(ETH Zürich)*
Numerical Analysis for Lévy Copula Processes

16:00 – 16:20 **Stanley Mathew** *(Frankfurt)*
Amortizing Forward: An Alternative Contract For Hedging Currency Risk

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16:30 – 16:50 **Christian Gründl** *(Heidelberg)*
Computation of Credit Risk Using Sparse Grids

17:00 – 17:20 **Marlkus Holtz** *(Bonn)*
Valuation of Performance-Dependent Options using Sparse Grids

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Vortragsauszüge

Kees Oosterlee *(Delft)*

Fast and Accurate Pricing of Early Exercise Options with the Fast Fourier Transform

When valuing and risk managing exotic derivatives, practitioners demand fast and accurate prices and sensitivities. Since the models being used in practice are becoming increasingly more complex, efficient methods have to be developed that can cope with the high dimensionality of such models. Aside from non-standard exotic derivatives, plain vanilla options in many stock markets are actually of the American type. As any pricing and risk management system has to be able to calibrate to these plain vanilla options, it is of the utmost importance to be able to value these American options quickly and accurately. In this paper we present highly efficient methods for options with early exercise or callable features. The methods developed rely heavily on the fast Fourier Transform (FFT). Early FFT based solution methods were mainly developed for pricing European options. Andricopoulos introduced the quadrature method QUAD, which can be used to value a variety of options with exotic features, assuming that the underlying process follows geometric Brownian motion. OSullivan combined the early Fourier transform methods with the QUAD method, and enabled the use of a fast resolution of the probability density function of the logarithm of the underlying. His method can be applied to a general set of underlying Lévy processes as well as to exotic path dependent features. Inspired by the quadrature pricing techniques, we propose a solution method that utilises the power of the FFT as much as possible. At the same time, we wish to get a grip on the resulting errors. The computational complexity in our recently developed solution methods reduces to $O(MN \log N)$, with M the number of observation times, and N the number of points in price dimension. In combination with Richardson extrapolation, and by using the fractional FFT method, we aim for the highest efficiency for the pricing of callable options under models where the underlying asset is exponentially affine in the state variables. This is the case for exponentially affine jump-diffusion models, exponential Lévy models, and variants hereof. We focus on both one-dimensional and two-dimensional processes, the latter class allowing us to develop efficient pricing methods for popular stochastic volatility models, such as the Heston model. The methods proposed are compared, also with iterative solution methods that solve the partial (integro-) differential equation with finite differences for a variety of American and Bermudan option pricing models with early exercise features.

Ansgar Jüngel *(Mainz)*

Nonlinear Black-Scholes-type Equations for Financial Derivatives

Standard financial derivatives like European options are priced by the famous Black-Scholes model which has the form of a linear parabolic equation. The Black-Scholes equation is derived under quite restrictive assumptions, e.g., no transaction costs occur and the market is complete. Without these conditions the resulting models may become nonlinear due to feedback effects, for instance.

In this talk some nonlinear Black-Scholes-type equations are discussed. The first model including the effect of transaction costs is a Black-Scholes equation with a volatility depending on the second derivatives of the solution. It has been derived by Barles and Soner in 1998. The equation is discretized using a higher order compact finite difference scheme and some numerical convergence results are given. The solutions are compared to those from the standard Black-Scholes model.

The second model describes the optimal value function in incomplete markets and has been derived by Leitner in 2001. It gives information on the transaction of shares the investor should make in order to maximise her or his profit. The market is allowed to possess non-tradable state variables like the employee income, weather parameter etc. Mathematically, the model is a parabolic equation with quadratic gradients. Existence and uniqueness results and numerical simulations are presented in order to show the influence of the nontradable state variables.

Ralf Forster *(FU Berlin)*

Fast and Reliable Pricing of American Options With Local Volatility

We consider the parabolic obstacle problem with variable coefficients appearing in the Black-Scholes equations with local volatility when evaluating American put options in mathematical finance. By suitable transformations we symmetrize the discretized problem to solve it by multigrid methods. Real-life data were used for the parameters and adapted carefully to the transformed problem. Finally we present numerical results to underline the convenience of this method within this setting.

Sebastian Quecke *(Köln)*
Pricing American Options using Quadrature

An efficient numerical method for pricing American-style options is presented. Starting from a discounted expectation approach we develop a flexible algorithm for option pricing. The computational complexity of the method is analyzed and error bounds are given. Adaptive quadrature and adaptive interpolation techniques allow to control accuracy. The resulting algorithm can be applied to fairly arbitrary market models and option types without considerable modifications. We present numerical results for several types of options under Lévy processes as example.

Erich W. Farkas *(ETH Zürich)*
Numerical Analysis for Lévy Copula Processes

We consider the valuation of derivative contracts on baskets where prices of single assets are Lévy like Feller processes of tempered stable type. The dependence among the marginals' jump structure is parametrized by a Lévy copula. For marginals of regular, exponential Lévy type in the sense of Boyarchenko and Levendorskii we show that the infinitesimal generator $\mathcal{A}$ of the resulting Lévy copula process is a pseudo-differential operator whose principal symbol is a distribution of anisotropic homogeneity.

We analyze the jump measure of the corresponding Lévy copula processes. We prove the domains of their infinitesimal generators $\mathcal{A}$ are certain anisotropic Sobolev spaces. In these spaces and for a large class of Lévy copulas, we prove a Garding inequality for $\mathcal{A}$.

We design a wavelet-based dimension-independent tensor product discretization for the efficient numerical solution of the parabolic Kolmogoroff equation $u_t + \mathcal{A}u = 0$ arising in valuation of derivative contracts under possibly stopped Lévy copula processes. We show that diagonal preconditioning yields bounded condition number of the resulting matrices.

Stanley Mathew *(Frankfurt)*
Amortizing Forward: An Alternative Contract For Hedging Currency Risk

With a classical forward contract investors protect their position against unfavorable developments of the currency in foreign exchange markets. In doing so, they are hedging market risk, while at the same time are – often regretted – giving up all chances of an enhanced performance should the market move in their favor. A contract that offers an opportunity to participate in the latter case is the Amortizing Forward introduced here. It provides the investor with a currency rate that protects his position, but enables a reduced liability.

The contract comprises an upper level K above which the notional amount is lowered (amortized) stepwise, and an upper barrier B, which terminates the Amortizing Forward entirely if triggered. An analytic solution is found for a simple case and it is demonstrated why for others only numerical methods can be considered. An unexpected result is exhibited by the fact that the barrier does not diminish but increase the value of the contract. This stands in contrast to the classical barrier-options, where the barrier is intended to reduce cost.

Christian Gründl *(Heidelberg)*
Computation of Credit Risk Using Sparse Grids

In den vergangenen 15 Jahren wurden fundierte Modelle zur Berechnung der Ausfallwahrscheinlichkeit einzelner Kreditkunden entwickelt. Anhand dieser Modelle wurden im Zuge des Internationalen Abkommens Basel II Richtlinien für die Eigenkapitalausstattung von Banken getroffen, die von bankinternen Ausfallwahrscheinlichkeiten abhängt.

Dabei werden heutzutage die meisten Berechnungen von korrelierten Ausfallwahrscheinlichkeiten mit Hilfe der Monte-Carlo Methode durchgeführt. In der Vergangenheit wurde gezeigt, dass man mit Hilfe stochastischer Differentialgleichungen Aufgabenstellungen der Optionspreis-Theorie modellieren kann. Diese Verfahren werden in dem Vortrag auf das Modell der Kreditrisikoberechnung übertragen. Einsatz findet hierbei ein Mehrgitter-Verfahren kombiniert mit der Technik der dünnen Gitter.

Markus Holtz *(Bonn)*

Valuation of Performance-Dependent Options using Sparse Grids

The efficient and accurate valuation of financial derivatives is a central topic in computational finance. Performance-dependent options are an important class of derivatives whose payoff depends on the performance of one asset in comparison to other assets. The fair price of such options can be determined by the martingale approach as a multidimensional integral whose dimension is the number of assets under consideration. Usually, the integrand is discontinuous, though, which makes accurate solutions difficult to achieve by numerical approaches.

For performance-dependent options, we derive a representation of the solution which only involves the evaluation of several multivariate normal distributions. This solution uses novel tools from computational geometry which faciliate the fast enumeration of all cells in a hyperplane arrangement and its orthant decomposition. We show that the arising normal distributions can be efficiently computed using sparse grid quadrature methods. This way, the complexity and the dimensionality of the integration problem can be significantly reduced which allows the efficient pricing of performance-dependent options even for large benchmarks, which is illustrated in several numerical examples.