Minisymposium 13

Approximationsmethoden für Probleme auf der Sphäre

Leiter des Symposiums:

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Die Approximationstheorie für Probleme auf sphärischen Geometrien hat in den letzten Jahren sehr aktuelle und interessante Weiterentwicklungen erfahren und vielfältige neue Anwendungen solcher Methoden sind studiert worden. Dies trifft besonders zu auf Aufgaben der Geomathematik wie die satellitengestützte Bestimmung des Gravitationsfeldes der Erde, die mathematischen Algorithmen für bildgebende Verfahren wie CT und MRI oder die elektronenmikroskopische Analyse von Makromolekülen und Kristallen. All diesen Anwendungen ist gemeinsam, dass die durch moderne Messverfahren gewonnenen riesigen Datenmengen von Funktionen auf der Sphäre effizient ausgewertet werden müssen. Die mathematischen Methoden, die hier eingesetzt werden und die wir diskutieren wollen, reichen von inversen Problemen, Multiskalenanalyse, Wavelets, radialen Basisunktionen, polynomialer Approximation bis hin zu Fehlerabschätzungen und schnellen Algorithmen.

Donnerstag, 21. September

Seminarraum 2, AVZ I, Endenicher Allee 11-13

15:00 – 15:50 **Matthias Holschneider** *(Potsdam)*
Wavelet Frames auf der 2-Sphäre: Konstruktion und Anwendung

16:00 – 16:20 **Dirk Langemann** *(Lübeck)*
Threedimensional approximation of the total ponderomotive force on round uncharged objects in an electric field

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16:30 – 16:50 **Martin Gutting** *(Kaiserslautern)*
The spherical Bernstein wavelet

17:00 – 17:50 **Volker Michel** *(Kaiserslautern)*
Die Modellierung und Approximation der Ausbreitung von Erdbebenwellen mittels Produktkernansätzen

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Freitag, 22. September

Seminarraum 2, AVZ I, Endenicher Allee 11-13

15:00 – 15:50 **Daniel Potts** *(Chemnitz)*
Fast summation of radial functions on the sphere

16:00 – 16:20 **Jens Keiner** *(Lübeck)*
Fast evaluation of quadrature formulae on the sphere

16:30 – 16:50 **Ralf Hielscher** *(Bergakademie TU Freiberg)*
Numerical inversion of the one–dimensional Radon transform on SO(3)

17:00 – 17:50 **Frank Filbir** *(Institute of Biomathematics and Biometry, GSF National Research Center, Neuherberg, Germany)*
Convolution structures and polynomial approximation on the sphere

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Vortragsauszüge

Matthias Holschneider *(Potsdam)*
Wavelet Frames auf der 2-Sphäre: Konstruktion und Anwendung

Wir konstruieren ein kontinuierliches Frame von Poisson Wavelets auf der 2-Sphäre. Diese erlauben es, Quadratische Formen mit polynomialem Symbol, wie sie als a priori Information in geomagnetischen Modellen auftauchen, durch Punktauswertungen zu berechnen. Des Weiteren zeigen wir, wie durch Diskretisierung ein diskretes Frame von sphärischen Wavelets kontruiert werden kann.

Dirk Langemann *(Lübeck)*
Threedimensional approximation of the total ponderomotive force on round uncharged objects in an electric field

Droplets on outdoor high-voltage equipment suffer a total ponderomotive force which is non-vanishing in general. We consider a model problem of a round uncharged test-body. We show that the total force can be given as a series of inhomogeneity indicators of the undisturbed electric field. While the series is derived rather easily in 2d, it involves interesting aspects of the spherical harmonics in the use of 3d Fourier techniques. The found series expansion establishes a relation between the solutions of two Poisson equations on different domains. It is found that the expansion converges fast. The results are applied for droplets on realistically shaped insulators.

Martin Gutting *(Kaiserslautern)*
The spherical Bernstein wavelet

In this work we introduce a new bandlimited spherical wavelet: The Bernstein wavelet. It possesses a couple of interesting properties. To be specific, we are able to construct bandlimited wavelets free of oscillations. The scaling function of this wavelet is investigated with regard to the spherical uncertainty principle, i.e., its localization in the space domain as well as in the momentum domain is calculated and compared to the well-known Shannon scaling function. Surprisingly, they possess the same localization in space although one is highly oscillating whereas the other one shows no oscillatory behavior. Moreover, the Bernstein scaling function turns out to be the first bandlimited scaling function known to the literature whose uncertainty product tends to the minimal value 1.

This is joint work with M. J. Fengler and W. Freeden.

Volker Michel *(Kaiserslautern)*
Die Modellierung und Approximation der Ausbreitung von Erdbebenwellen mittels Produktkernansätzen

Das elastische Verhalten der Erde lässt sich näherungsweise durch die Cauchy–Navier–Gleichung beschreiben. Für ein relativ einfaches Erdmodell ist hier ein Lösungssystem im Frequenzraum aus so genannten Hansenvektoren bekannt. Damit können normale Erdbebenwellen aber auch Eigenschwingungen, wie sie nach sehr schweren Beben auftreten, modelliert werden. In dem Vortrag wird nach der Einführung in diesen bekannten Teil der Theorie gezeigt, dass mittels einer inversen Fouriertransformation ein Orthonormalsystem von zeit- und ortsabhängigen Funktionen berechnet werden kann. Diese dienen dann dazu, um Produktkerne zu definieren, auf deren Basis spezielle Spline- und Waveletverfahren entwickelt werden können. Der Vorteil dieser Approximationsmethoden liegt in ihrer Lokalisierung. Als Anwendung wird die Approximation von sich ausbreitenden Erdbebenwellen auf der Basis zeitlich und räumlich diskreter Verschiebungswerte (Seismogramme) auf der (näherungsweise) sphärischen Oberfläche numerisch untersucht. Als Daten werden eine einfache Referenzwelle und eine realistische Simulation eines Bebens in Japan verwendet.

P. Kammann, V. Michel: Time–Dependent Cauchy–Navier Splines and their Application to Seismic Wave Front Propagation, *Schriften zur Funktionalanalysis und Geomathematik*, Preprint Nr. 26 (2006), TU Kaiserslautern;
V. Michel: Theoretical Aspects of a Multiscale Analysis of the Eigenoscillations of the Earth, *Revista Matematica Complutense*, 16 (2003), 519-554.

Daniel Potts *(Chemnitz)*
Fast summation of radial functions on the sphere

Radial functions are a powerful tool in many areas of multidimensional approximation, especially when dealing with scattered data. We present a fast approximate algorithm for the evaluation of linear combinations of radial functions on the sphere $\mathbb{S}^2$. The approach is based on a particular rank approximation of the corresponding Gram matrix and fast algorithms for spherical Fourier transforms. The proposed method takes $\mathcal{O}(L)$ arithmetic operations for L arbitrarily distributed nodes on the sphere. In contrast to other methods, we do not require the nodes to be sorted or pre-processed in any way, thus the pre-computation effort only depends on the particular radial function and the desired accuracy. We establish explicit error bounds for a range of radial functions and provide numerical examples covering approximation quality, speed measurements, and a comparison of our particular matrix approximation with a truncated singular value decomposition.

This talk is based on joint work with J. Keiner (University of Lübeck) and S. Kunis (Chemnitz University of Technology).

Jens Keiner *(Lübeck)*
Fast evaluation of quadrature formulae on the sphere

Recently, a fast approximate algorithm for the evaluation of expansions in terms of standard $\mathrm{L}^2(\mathbb{S}^2)$-orthonormal spherical harmonics at arbitrary nodes on the sphere $\mathbb{S}^2$ has been proposed in $[2]$. Our aim is to develop a fast algorithm for the adjoint problem, hence the computation of expansion coefficients from sampled data by means of quadrature rules.
We give a formulation in matrix-vector notation and an explicit factorisation of the corresponding spherical Fourier matrix that is based on the first algorithm. Starting from this factorisation, we obtain the corresponding adjoint factorisation and are able to implement the corresponding transform. This 'adjoint' algorithm can be employed to evaluate quadrature rules for arbitrary quadrature nodes and weights on the sphere $\mathbb{S}^2$.
We provide results of test computations with respect to stability, accuracy and performance of the obtained algorithm. As examples, we consider a variety of proposed test functions using classical Gauß-Legendre and Clenshaw-Curtis quadrature rules. Furthermore, we also consider an equidistribution from $[3]$ and the HEALPix pixelation scheme ($[1]$), each with equal weights for all nodes to obtain a convenient quadrature rule. Especially the HEALPix scheme has great relevance as data storage standard in certain applications like cosmic microwave background estimation.

References:

[1] K. M. Górski, E. Hivon, A. J. Banday, B. D. Wandelt, F. K. Hansen, M. Reinecke, and M. Bartelmann. Healpix: A framework for high-resolution discretization and fast analysis of data distributed on the sphere. *The Astrophysical Journal*, 622:759–771, 2005.

[2] S. Kunis and D. Potts. Fast spherical Fourier algorithms. *J. Comput. Appl. Math.*, 161:75 – 98, 2003.

[3] W. Freeden, T. Gervens, and M. Schreiner. *Constructive Approximation on the Sphere*. Oxford University Press, Oxford, 1998.

Ralf Hielscher *(Bergakademie TU Freiberg)*
Numerical inversion of the one–dimensional Radon transform on SO(3)

We are concerned with the numerical inversion of the one–dimensional Radon transform on the rotational group SO(3) subject to a non–negativity constraint. While the Radon transform on $\mathbb{R}^3$ attracted much attention during the last fifteen years due to its connection to tomography the Radon transform on SO(3) did not. Our problem has practical applications in texture analysis, i.e. the analysis of crystallographic preferred orientation in polycrystalline materials as metals or rocks. We characterize the Radon transform on SO(3) as an operator between Sobolev spaces and present a spline based inversion algorithm that is especially well suited for scattered data as they are provided by the application in mind. A core item of our algorithm is the fast non–uniform spherical Fourier transform.

Additionally, we introduce a framework that allows for some basic error estimates of the inverse transform.

Frank Filbir *(Institute of Biomathematics and Biometry, GSF National Research Center, Neuherberg, Germany)*
Convolution structures and polynomial approximation on the sphere

We introduce convolution structures on $\mathbb{N}_0$ and on the intervall $[-1,1]$ established by so-called product formulas for orthogonal polynomials. After that we will show how to use these structures in order to construct good kernels. Due to the polynomial preproduction property of the associated approximation process the de la Vallée Poussin kernel for

Jacobi expansions is of special interest. This operator is then used to approximate functions f defined on the unit sphere $S^d \subset \mathbb{R}^{d+1}$, using samples of f at scattered sites. We are going to show how to obtain so-called Marcinkiewicz-Zygmund inequalities and establish concrete error estimates. Finally we show how to use these results in order to derive positive quadrature rules on S^d of high accuracy and based on function values at scattered point on S^d.