Minisymposium 1

Discrete Optimization

Leiter des Symposiums:

Prof. Dr. Jens Vygen
Forschungsinstitut für
Diskrete Mathematik
Universität Bonn
Lennestraße 2
53113 Bonn, Germany

Prof. Dr. Robert Weismantel
Fakultät für Mathematik / IMO

Otto-von-Guericke-Universität Magdeburg
Universitätsplatz 2
39106 Magdeburg, Germany

Dienstag, 19. September

Übungsraum 3, Geographisches Institut, Meckenheimer Allee 166

15:00 – 15:50 **Martin Skutelle** *(Dortmund)*
Network flows with path restrictions

16:00 – 16:20 **Christoph Buchheim** *(Köln)*
Compact IP formulations of Boolean optimization problems

16:30 – 16:50 **Dennis Michaels** *(Magdeburg)*
Discrete methods for tackling nonlinear mixed integer optimization problems in chemical engineering

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17:00 – 17:50 **Alexander Martin** *(Darmstadt)*
Minimizing switching networks

Mittwoch, 20. September

Übungsraum 3, Geographisches Institut, Meckenheimer Allee 166

15:00 – 15:50 **Andreas Brieden** *(München)*
Optimization for agriculture

16:00 – 16:20 **Marc Pfetsch** *(Berlin)*
Symmetry reduction in graph coloring via orbitopes

16:30 – 16:50 **Ulrich Brenner** *(Bonn)*
A faster polynomial-time algorithm for the unbalanced Hitchcock transportation problem

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17:00 – 17:50 **Fritz Eisenbrand** *(Dortmund)*
The stable set polytope of quasi-line graphs

Vortragsauszüge

Martin Skutelle *(Dortmund)*

Network Flows with Path Restrictions

Network flows are usually specified by assigning flow values to the arcs of a given network. Flow conservation constraints ensure that, except for certain source and sink nodes, the amount of flow entering a node equals the amount of flow leaving that node. Moreover, arc capacities often bound the flow values on the arcs. It is a classical and well known result that any flow can be decomposed into flows along paths and cycles. This insight yields the so-called path-based formulation of a flow problem where the task is to assign flow values to paths and cycles (instead of arcs). By imposing certain restrictions on the paths that are used to send flow, interesting variants of classical flow problems can be derived. Motivated by practical applications, one can, for example, bound

- the number of flow-carrying paths (*unsplittable* or k-*splittable* flow problems),
- the length of flow-carrying paths (*length-bounded* flow problems),
- the amount of flow sent through each path (flows with *path capacities*).

Also all sorts of combinations of the above mentioned constraints can be taken into consideration. We present recent results on network flows with path restrictions and discuss interesting directions for future research.

Christoph Buchheim *(Köln)*

Compact IP formulations of Boolean optimization problems

We present a new polyhedral approach to general logic optimization problems. Compared to other methods, our approach produces much smaller IP models, making it more efficient from the practical point of view. We mainly obtain this by two different ideas: first, we do not require the objective function to be in any normal form. The transformation into a normal form usually leads to the introduction of many additional variables or constraints. Second, we reduce the problem to the degree-two case in a very efficient way, using a slightly extended formulation. The resulting model turns out to be closely related to the max-cut problem; we can show that the corresponding polytope is a face of a suitable cut polytope. In particular, our separation problem can be completely reduced to the separation problem for max-cut. Experimental results show that our branch-and-cut implementation of this approach is significantly faster than other methods.

Dennis Michaels *(Magdeburg)*
Discrete methods for tackling nonlinear mixed integer optimization problems in chemical engineering

Joint work with Utz-Uwe Haus and Robert Weismantel

Many optimization problems in chemical engineering give rise to non-convex nonlinear mixed-integer optimization problems. While many tools for generating locally optimal solutions are nowadays available, determining a globally optimal solution still remains a challenging task. In this talk an approach to tackle such instances is introduced that mostly resorts to techniques from discrete optimization. A hierarchy of mixed-integer linear problems is defined that contain all solutions of the original instance. For this, the nonlinear terms occuring in the original formulation are polyhedrally relaxed respecting local and global properties in the domain. A key step of this approach is to identify combinatorial substructures like stable sets or nonlinear flow conservation conditions that are given by the constraints of the original nonlinear model. A linear description of those substructures leads to valid inequalities that strengthen the linear relaxations. The capability of this approach is demonstrated by considering two different applications coming from chemical engineering.

Alexander Martin *(Darmstadt)*
Minimizing switching networks

Switching networks are directed acyclic graphs with specified disjoint sets of input and output nodes. A connection request is a partial function from the set of output nodes to the set of input nodes specifying which input node needs to be routed to which output nodes. A switching network is called rearrangeably nonblocking with respect to multicast traffic if all connection requests are routable, that is, if for each request there exists a set of mutually vertex-disjoint directed trees connecting each input node to its designated output nodes. Clos networks are switching networks, where nodes are set up in stages to reduce size and cost.
The problem of characterizing routability of multicast-rearrangeable Clos networks is still open. In this talk we bring some new insights into this problem. We formulate the

problem as a vertex-coloring problem. We identify critical requests whose routability implies the routability of all others by applying some known theorems, by using reduction techniques and by exploiting symmetry. By efficiently enumerating all critical requests, we are able to characterize the routability of Clos networks with up to 32 input and output nodes.

Andreas Brieden *(München)*
Optimization for agriculture

In many regions farmers cultivate a number of small lots that are distributed over a wider area. This leads to high overhead costs and economically prohibits use of high tech machinery hence results in a non-favorable cost-structure of production. The classical form of land consolidation is typically too expensive and too rigid, whence consolidation based on lend-lease agreements has been suggested. In order to exploit the potential of this method specific mathematical optimization algorithms have to be developed that use many ideas from different mathematical areas. E.g., one approach leads to a quadratic optimization model that can be approximately solved by means of the theory of convex bodies. In subroutines Hadamard matrices are required to create mixed integer linear programs whose "good nature" can be proved by a careful analysis of the simplex algorithm.

Marc Pfetsch *(Berlin)*
Symmetry reduction in graph coloring via orbitopes

It is a well-known phenomenon that symmetries in integer programming (IP) formulations makes them extremely hard to solve. The reason is that, on the one hand, the corresponding LP relaxations become poor and, on the other hand, the branch-and-bound tree is unnecessarily large, because equivalent solutions are found again and again. This situation also occurs for a natural IP formulation for *graph coloring*, which has a variable for each pair of a node in the graph and a potential color.
We have recently introduced so-called partitioning *orbitopes*, which are the convex hulls of 0/1 matrices with exactly one 1 in each row, whose columns are lexicographically sorted. The goal is to remove symmetry by isolating a lexicographically maximal representative in each orbit of the full symmetric group acting on the columns of the matrices. We derived a complete linear description of these polytopes. In this talk, we investigate

the integer hulls of the intersections of orbitopes with the polytopes associated with the mentioned graph coloring formulation. The goal is to remove symmetry as much as possible. It turns out that even describing the dimensions of the resulting polytopes seems to be quite complicated (and so is the investigation of the facial structures). Nevertheless, we will discuss several classes of valid inequalities and present computational results based on them.

Joint work with Yuri Faenza and Volker Kaibel.

Ulrich Brenner *(Bonn)*

A faster polynomial-time algorithm for the unbalanced Hitchcock transportation problem

We present a new algorithm for the Hitchcock transportation problem. On instances with n sources and k sinks, our algorithm has a worst-case running time of $O(nk^2(\log n + k \log k))$. This algorithm closes a gap between algorithms which have a running time linear in n but exponential in k and a polynomial-time algorithm with running time $O(nk^2 \log^2 n)$.

Fritz Eisenbrand *(Dortmund)*

The stable set polytope of quasi-line graphs

It is a long standing open problem to find an explicit description of the stable set polytope of *claw-free graphs*. Yet more than 20 years after the discovery of a polynomial algorithm for the maximum stable set problem for claw-free graphs, there is even no conjecture at hand today.
Such a conjecture exists for the class of *quasi-line graphs*. This class of graphs is a proper superclass of line graphs and a proper subclass of claw-free graphs for which it is known that not all facets have $0/1$ normal vectors. The *Ben Rebea conjecture* states that the stable set polytope of a quasi-line graph is completely described by *clique-family* inequalities. Chudnovsky and Seymour recently provided a decomposition result for claw-free graphs and proved that Ben Rebea's conjecture holds, if the quasi-line graph is not a *fuzzy circular interval graph*.
In this talk I present a proof of the Ben Rebea conjecture by showing that it also holds for fuzzy circular interval graphs.

Joint work with G. Oriolo, G.Stauffer and P. Ventura.