Minisymposium 10 - The use of proof theory in mathematics

The Elimination of Prime Ideals

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Contrarily to André Weil, who wanted to "eliminate the elimination", i.e. eliminate computations, we think that it is important to use abstract settings in order to make computations. Unfortunately the mathematicians who invented the abstract objects did not always tell us which "concrete objects" they started from. One reason is that they had a direct intuition of these objects. Without being able to explain the mystery, we nevertheless propose concrete objects as supports of abstract ones. Here we try to understand how prime ideals can be eliminated from abstract proofs in order to obtain constructive proofs using concrete substitutes for prime ideals.

For short, when a generic prime ideal is used in order to get an algebraic result, we can often understand this machinery as a way to prove that a certain ring is trivial. The proof is ad absurdum: if the ring were not trivial, then a prime ideal would exist in this ring. There are two variants: first, if the ring were not trivial, then a maximal prime ideal would exist in this ring; secondly, if the ring were not trivial, then a minimal prime ideal would exist in this ring. We will explain the computations that are involved when deciphering several classical uses of prime ideals.